Higher Order Methods for Incompressible Fluid Flow
by Deville, Fischer and Mund, Cambridge University Press, 499 pp.

Although the title suggests that this work might cover a wider range of material, this book is actually a monograph almost entirely devoted to an exposition of finite element and spectral element methods for incompressible flow problems, with emphasis on the latter. Spectral, finite difference, and finite volume methods are given only passing mention. Higher order methods are indeed given a great deal of space but the full range of methods is discussed so this work might better be considered a general text on finite and spectral element methods.

When solutions of high accuracy are required, one has to choose between high order methods and low order ones with a larger number of grid points. The former are more expensive in terms of the cost per degree of freedom but this is more than compensated for by the reduction in the number of variables that need to be computed. The problems with high order methods (and the primary reason why that have not seen more use) are associated with the difficulty of enforcing the boundary conditions and of dealing with complex geometry.

This book is written in the mathematical style often adopted by finite element practitioners but it is accessible to engineers. Some discussion of fluid mechanics from a physical point of view is included and the presentation covers everything from the mathematical underpinnings of the methods to some details of programming. It is well organized, progressing from simple problems to more complex ones in an orderly manner. Finally, it is well written and easy to follow.

The initial chapter is on fluid mechanics and is probably too short for anyone not already familiar with the subject but it does put some of the issues into perspective for those that have a background in the field. The last part of the chapter covers issues connected with computing in a similar style.

The second chapter presents basic methods for elliptic (boundary value) problems in one dimension. It is a good general introduction to the major concepts associated with finite element methods including the Galerkin, Petrov–Galerkin, collocation and spectral element techniques and the h- and p-types of methods and error analysis. It ends with a short presentation of iterative solution methods, ending with the conjugate gradient and related methods.

Chapter 3 presents an introduction to the treatment of unsteady problems: hyperbolic and parabolic problems in one spatial dimension. It starts with a short overview of methods for initial value problems including the standard methods and splitting or fractional step methods. It then deals with unsteady diffusion and convection and the combination of the two. It finishes with a treatment of Burgers equation and the subcycling or iterative methods needed to solve to find the solution at each time step when implicit methods are applied to nonlinear problems. A numerical example is given which nicely illustrates the properties of the methods.

Methods for problems in more than one space dimension are introduced in Chapter 4, starting with rectangular geometry for which tensor products of the elements introduced for one-dimensional problems may be used. It then goes on to the treatment of deformed geometries through the use of mappings and spectral element methods. The methods are then applied to diffusion and advection–diffusion problems. The nature of the matrices produced, their eigenvalues and iterative methods for solving the discretized problems are also discussed.

Solutions of the Stokes and Navier–Stokes equations are taken up in Chapter 5. The emphasis is again on spectral element methods. Considerable attention is given to the oscillations in the pressure that can result from methods using collocated arrangements of variables and means of curing them, principally through the use of staggered variable arrangements. A number of examples, all of which are in domains whose boundaries are coordinate lines in a Cartesian system are given.

Solutions of the unsteady versions of the same equations are discussed in the following chapter. The standard methods of updating the momentum equations and solving a Poisson equation for the pressure are given. The chapter then goes on to projection methods, including several variants. Finally, arbitrary Lagrangian–Eulerian (ALE) methods are considered and applied to free surface flows. The chapter ends with a number of examples illustrating the application of the methods described in the chapter.

Chapter 7 is devoted to domain decomposition and related topics. The purpose of domain decomposition is to make the task of creating grids easier and to allow programming on parallel computers. The principal means of solving such problems is through the use of Schwarz methods with overlap. These are described and their convergence and preconditioning are discussed. Then the spectral element multigrid and mortar element methods are described. The chapter ends with a few applications.

The final chapter is devoted to vector and parallel computers and the implementation of the methods on these machines. It gives descriptions of the machine architectures and some techniques for obtaining high performance from them.

There are several appendices devoted mainly to the exposition of a number of mathematical details.

As already noted, this book is quite well written and easy to follow. It is the only work that I am aware of that presents the spectral element method in detail. However, it also has some limits, as any work must. In particular, what is missing in this work is anything more than a passing mention of non-quadrilateral elements, grid generation for complex geometry, or the issues of dealing with unstructured grids. These are all important parts of finite element technology and one wishes they had been included. The description of methods and elements for three-dimensional problems is also extremely limited. Although a number of examples are given in most of the chapters, students would benefit from the inclusion of some examples in which all of the details of how the problem is set up and programmed. Despite these drawbacks, the book is a welcome addition to the literature in the field and is especially recommended to anyone that would like to learn about spectral element techniques.

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10.1016/S0997-7546(02)00007-9